

COLBY 00-02

IUHET 419

January 2000

SEARCHING FOR LORENTZ VIOLATION IN THE GROUND STATE OF HYDROGEN ¹

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INTRODUCTION

The hydrogen atom has a rich history as a testing ground of fundamental physics where small differences between theory and experiment have led to major advances [1]. With the advent of optical high-resolution spectroscopy and tunable dye lasers, new tests of quantum electrodynamics in hydrogen have become possible. The two-photon 1S-2S transition is especially suitable for high-precision tests and metrology because of its small natural linewidth of only 1.3 Hz. This transition has been measured in a cold atomic beam of hydrogen [2] with a precision of 3.4 parts in 10^{14} . It has also been observed in trapped hydrogen [3] with a precision of about one part in 10^{12} . As experimental techniques advance, the measurement of the line center to one part in 10^3 becomes plausible with an ultimate resolution of one part in 10^{18} , making new tests of fundamental theory possible.

The recent production of antihydrogen in experiments [4] ushers in a new era for testing fundamental physics by allowing direct high-precision comparisons of

¹Presented by R.B. at Orbis Scientiae 1999, Ft. Lauderdale, Florida, December 1999

hydrogen and antihydrogen [5]. Since the CPT theorem predicts that all local relativistic quantum field theories of point particles are invariant under the combined operations of charge conjugation C, parity reversal P, and time reversal T [6, 7], comparisons of the 1S-2S transition in hydrogen and antihydrogen should provide a new high-precision test of CPT. Indeed, two future experiments at CERN [8] are aimed at making high-resolution spectroscopic comparisons of the 1S-2S transitions in spin-polarized hydrogen and antihydrogen confined within a magnetic trap. The comparisons of the 1S-2S transition should have relative figures of merit comparable to that of the neutral meson system, which places a bound on the mass difference between the K_0 and \bar{K}_0 at less than 2 parts in 10^{18} [9].

In this proceedings, we first review a recent theoretical analysis we made of CPT and Lorentz tests in hydrogen and antihydrogen, which was published in Ref. [10]. This included investigations of on-going experiments in hydrogen as well as the proposed experiments at CERN comparing hydrogen and antihydrogen. We showed that these experiments can provide tests of both CPT-preserving and CPT-violating Lorentz symmetry. In addition to examining comparisons of 1S-2S transitions, we suggested other possible experimental signatures that are sensitive to CPT or Lorentz breaking, including measurements of the Zeeman hyperfine levels in the ground state of hydrogen. Some of these measurements are currently being made and preliminary results are presented for the first time in Walsworth's talk [11].

THEORETICAL FRAMEWORK

Our analysis uses a theoretical framework that describes CPT- and Lorentz-violating effects in an extension of the $SU(3) \times SU(2) \times U(1)$ standard model and quantum electrodynamics (QED) [12]. The framework originates from the idea of spontaneous CPT and Lorentz breaking in a more fundamental theory such as string theory [13, 14]. Within this framework, possible violations of CPT and Lorentz symmetry are included which maintain desirable features of quantum field theory, including gauge invariance, power-counting renormalizability, and microcausality. The model is highly constrained, and only a small number of terms are possible. These terms are controlled by parameters that can be bounded by experiments. This framework has been used to analyze neutral-meson experiments [13, 15, 16, 17], baryogenesis [18], photon properties [12, 19], Penning-trap experiments [20, 21, 22], atomic clock comparisons [23], muon experiments [24], and experiments in spin-polarized matter [25].

To investigate experiments in hydrogen and antihydrogen, it suffices to work in the context of the QED extension. The modified Dirac equation for a four-component spinor field ψ describing electrons and positrons of mass m_e and charge

$q = -|e|$ in a Coulomb potential A^μ is

$$\left(i\gamma^\mu D_\mu - m_e - a_\mu^e \gamma^\mu - b_\mu^e \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu}^e \sigma^{\mu\nu} + i c_{\mu\nu}^e \gamma^\mu D^\nu + i d_{\mu\nu}^e \gamma_5 \gamma^\mu D^\nu \right) \psi = 0 \quad . \quad (1)$$

Here, natural units with $\hbar = c = 1$ are used, $iD_\mu \equiv i\partial_\mu - qA_\mu$, and $A^\mu = (|e|/4\pi r, 0)$. The two terms involving the effective coupling constants a_μ^e and b_μ^e violate CPT, while the three terms involving $H_{\mu\nu}^e$, $c_{\mu\nu}^e$, and $d_{\mu\nu}^e$ preserve CPT. All five of these terms break Lorentz invariance. Since no CPT or Lorentz violation has been observed, these parameters are assumed to be small. Free protons are also described by a modified Dirac equation involving the corresponding parameters a_μ^p , b_μ^p , $H_{\mu\nu}^p$, $c_{\mu\nu}^p$, and $d_{\mu\nu}^p$.

A perturbative treatment in the context of relativistic quantum mechanics is used to examine the bound states of hydrogen and antihydrogen. In this approach, the unperturbed hamiltonian \hat{H}_0 and its energy eigenfunctions are the same for hydrogen and antihydrogen. All of the perturbations in free hydrogen described by conventional quantum electrodynamics are identical for both systems. However, the interaction hamiltonians for hydrogen and antihydrogen including the effects of possible CPT- and Lorentz-breaking are not the same. These are obtained in several steps [21], involving charge conjugation to obtain the Dirac equation for antihydrogen, a field redefinition to eliminate additional time derivatives in the Dirac equation, and the use of standard relativistic two-fermion techniques [26].

EXPERIMENTS WITH FREE HYDROGEN

We first consider free hydrogen and antihydrogen in the absence of external trapping potentials. Using a description in terms of the basis states $|m_J, m_I\rangle$, with $J = 1/2$ and $I = 1/2$ describing the uncoupled atomic and nuclear angular momenta, the leading-order energy corrections can be computed. The energy shifts at the 1S and 2S levels are found to be the same. For hydrogen they are given by

$$\Delta E^H(m_J = \pm \frac{1}{2}, m_I = \pm \frac{1}{2}) = (a_0^e + a_0^p - c_{00}^e m_e - c_{00}^p m_p) + \frac{m_J}{|m_J|} (-b_3^e + d_{30}^e m_e + H_{12}^e) + \frac{m_I}{|m_I|} (-b_3^p + d_{30}^p m_p + H_{12}^p) \quad , \quad (2)$$

where m_e and m_p are the electron and proton masses, respectively. The corresponding energy corrections for the 1S and 2S states of antihydrogen $\Delta E^{\bar{H}}$ are obtained from these by letting $a_\mu \rightarrow -a_\mu$, $d_{\mu\nu} \rightarrow -d_{\mu\nu}$, and $H_{\mu\nu} \rightarrow -H_{\mu\nu}$ for both the electron-positron and proton-antiproton coefficients.

The hyperfine interaction couples the electron and proton or positron and antiproton spins. The appropriate basis states are then $|F, m_F\rangle$ which are linear combinations of the states $|m_J, m_I\rangle$. The selection rules for the two-photon 1S-2S transition are $\Delta F = 0$ and $\Delta m_F = 0$. These selection rules require that the 1S-2S

transitions in free hydrogen and antihydrogen occur between states of the same spin configurations. As a result, the leading-order energy shifts are equal, and there are no observable leading-order shifts in frequency in either hydrogen or antihydrogen.

There are, however, subleading-order shifts in the 1S-2S frequencies. These are due to small relativistic corrections of order α^2 times the CPT- or Lorentz-breaking parameters which are different at the 1S and 2S levels. For example, the term proportional to b_3^e results in a frequency shift in the $m_F = 1 \rightarrow m_{F'} = 1$ transition relative to that of the $m_F = 0 \rightarrow m_{F'} = 0$ line (which remains unshifted) equal to $\delta\nu_{1S-2S}^H \approx -\alpha^2 b_3^e / 8\pi$. However, electron bounds obtained in $g-2$ experiments [22] suggest that b_3^e is sufficiently small so that $\delta\nu_{1S-2S}^H$ would be below the expected 1S-2S line resolution.

EXPERIMENTS WITH TRAPPED HYDROGEN

The experiments to be performed at CERN will use trapped hydrogen and antihydrogen in a magnetic field B . We use the conventional labels $|a\rangle_n$, $|b\rangle_n$, $|c\rangle_n$, and $|d\rangle_n$ in order of increasing energy to denote the four S-state hyperfine levels of hydrogen with principal quantum number n . The $|b\rangle_n$ and $|d\rangle_n$ states have proton and electron spins that are aligned, while the remaining two states have mixed spin configurations given by

$$|c\rangle_n = \sin\theta_n |-\frac{1}{2}, \frac{1}{2}\rangle + \cos\theta_n |\frac{1}{2}, -\frac{1}{2}\rangle \quad , \quad (3)$$

$$|a\rangle_n = \cos\theta_n |-\frac{1}{2}, \frac{1}{2}\rangle - \sin\theta_n |\frac{1}{2}, -\frac{1}{2}\rangle \quad . \quad (4)$$

The mixing angles depend on n and obey $\tan 2\theta_n \approx (51 \text{ mT})/n^3 B$.

The states $|c\rangle_n$ and $|d\rangle_n$ are low-field seeking states that remain confined in the trap. However, collisional effects lead to a loss of population over time of the $|c\rangle_n$ states. One possible measurement would therefore be to compare the frequencies ν_d^H and $\nu_d^{\bar{H}}$ for transitions between $|d\rangle_n$ states at the 1S and 2S levels. These measurements are particularly attractive because the 1S-2S $|d\rangle_1 \rightarrow |d\rangle_2$ transitions are field-independent for small values of B . However, since the spin configurations of the 1S $|d\rangle_1$ and 2S $|d\rangle_2$ states are the same, we find no observable frequency shifts to leading order in this case, i.e., $\delta\nu_d^H = \delta\nu_d^{\bar{H}} \simeq 0$.

An alternative experiment would look at transitions involving the mixed states $|c\rangle_n$ and $|a\rangle_n$. Here, the n dependence in the hyperfine splitting leads to a difference in the amount of spin mixing at the 1S and 2S levels. This gives rise to a nonzero frequency shift in 1S-2S transitions between $|c\rangle_n$ hyperfine states:

$$\delta\nu_c^H \simeq -(\cos 2\theta_2 - \cos 2\theta_1)(b_3^e - b_3^p - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p) \quad , \quad (5)$$

The corresponding transition for antihydrogen can be computed as well. The hyperfine states in antihydrogen in the same magnetic fields have opposite spin assignments for the positron and antiproton compared to those of the electron and proton in hydrogen. The resulting shift $\delta\nu_c^{\bar{H}}$ for antihydrogen is the same as for hydrogen except that the signs of b_3^e and b_3^p are changed.

Two possible experimental signatures for CPT and Lorentz breaking follow from these results. The first involves looking for sidereal time variations in the frequencies ν_c^H and $\nu_c^{\bar{H}}$. The second involves measuring the instantaneous 1S-2S frequency difference in hydrogen and antihydrogen in the same magnetic trapping fields. In either case, the strength of the signal would depend on the difference in the amount of spin mixing at the 1S and 2S levels. The optimal experiment would be one that maximizes the 1S-2S spin-mixing difference, which is controlled by the magnetic field B . Since the 1S-2S $|c\rangle_1 \rightarrow |c\rangle_2$ transition in hydrogen and antihydrogen is field dependent, these experiments would need to overcome line broadening effects due to field inhomogeneities in the trap.

EXPERIMENTS ON THE GROUND-STATE HYPERFINE LEVELS

The best tests of CPT and Lorentz symmetry in atomic systems are those that have the sharpest frequency resolutions. It is therefore natural to consider other transitions in hydrogen and antihydrogen besides the 1S-2S transition that can be measured with high precision. One candidate set involves measurements of the ground-state hyperfine levels in hydrogen and antihydrogen. For example, hydrogen maser transitions between $F = 0$ and $F' = 1$ hyperfine states can be measured with accuracies of less than 1 mHz. High-resolution radio-frequency measurements can also be made on transitions between Zeeman hyperfine levels in a magnetic field.

To examine these types of experiments, we compute the energy shifts of the four hydrogen ground-state hyperfine levels in a magnetic field. The spin-dependent contributions to the energy are

$$\Delta E_a^H \simeq \hat{\kappa}(b_3^e - b_3^p - d_{30}^e m_e + d_{30}^p m_p - H_{12}^e + H_{12}^p) \quad , \quad (6)$$

$$\Delta E_b^H \simeq b_3^e + b_3^p - d_{30}^e m_e - d_{30}^p m_p - H_{12}^e - H_{12}^p \quad , \quad (7)$$

$$\Delta E_c^H \simeq -\Delta E_a^H \quad , \quad (8)$$

$$\Delta E_d^H \simeq -\Delta E_b^H \quad , \quad (9)$$

where $\hat{\kappa} \equiv \cos 2\theta_1$.

In a very weak or zero magnetic field $\hat{\kappa} \simeq 0$ and the energies of the states $|a\rangle_1$ and $|c\rangle_1$ are unshifted while the states $|b\rangle_1$ and $|d\rangle_1$ acquire equal and opposite shifts. The degeneracy of the three $F = 1$ levels is therefore lifted. A conventional hydrogen

maser operates on the field-independent transition $|c\rangle_1 \rightarrow |a\rangle_1$ in the presence of a small ($B \lesssim 10^{-6}$ T) magnetic field. Since $\hat{\kappa} \lesssim 10^{-4}$ in this case, the leading-order effects due to CPT and Lorentz violation are suppressed. However, the frequencies of the Zeeman hyperfine transitions between $F = 1$ levels are affected by CPT and Lorentz violation and have unsuppressed corrections. For example, the correction to the $|c\rangle_1 \rightarrow |d\rangle_1$ transition frequency in a very weak field is given by

$$\delta\nu_{c \rightarrow d}^{\text{H maser}} \simeq (-b_3^e - b_3^p + d_{30}^e m_e + d_{30}^p m_p + H_{12}^e + H_{12}^p)/2\pi \quad . \quad (10)$$

A signature of CPT and Lorentz violation would thus be sidereal time variations in the frequency $\nu_{c \rightarrow d}^{\text{H maser}}$.

The transition $|c\rangle_1 \rightarrow |d\rangle_1$ in a hydrogen maser is field-dependent, and one would expect field broadening to limit the resolution of frequency measurements. However, as described by Walsworth [11], it is possible to perform a double-resonance experiment [27] in which variations of the $|c\rangle_1 \rightarrow |d\rangle_1$ transition are determined by monitoring their effect on the usual $|a\rangle_1 \rightarrow |c\rangle_1$ maser line. This then permits a search for sidereal variations in the frequency $\nu_{c \rightarrow d}^{\text{H maser}}$. Walsworth's group at the Harvard-Smithsonian Center has begun this experiment, and their preliminary results indicate that the sidereal variations in $\nu_{c \rightarrow d}^{\text{H maser}}$ can be bounded at a level of approximately 0.7 mHz. This corresponds to a bound on the combination of parameters in $\delta\nu_{c \rightarrow d}^{\text{H maser}}$ in Eq. (10) at a level of 10^{-27} GeV. Defining a figure of merit as the ratio of the amplitude of the sidereal variations of the energy relative to the energy itself, i.e., $r_{\text{hf}}^H \equiv (\Delta E_{\text{hf}})_{\text{sidereal}}/E_{\text{hf}}$, one obtains from the results of Walsworth's experiment the value

$$r_{\text{hf}}^H \lesssim 10^{-27} \quad . \quad (11)$$

This now gives one of the sharpest bounds on CPT and Lorentz violation for protons and electrons.

In principle, measurements of this kind can also be made on the Zeeman hyperfine levels in antihydrogen. Since only in a direct comparison of matter and antimatter can the CPT-violating effects be isolated, it is hoped that the technical obstacles of performing radio-frequency spectroscopy in trapped antihydrogen can be overcome. As an alternative to measurements in a very weak magnetic field, which might be hard to maintain in a trapping environment, one could perform a comparison of $|c\rangle_1 \rightarrow |d\rangle_1$ transitions in hydrogen and antihydrogen at the field-independent transition point $B \simeq 0.65$ T. At this field strength, the electron and proton spins in the $|c\rangle_1$ state are highly polarized with $m_J = \frac{1}{2}$ and $m_I = -\frac{1}{2}$. The transition $|c\rangle_1 \rightarrow |d\rangle_1$ is effectively a proton spin-flip transition. The instantaneous difference in this transition for hydrogen and antihydrogen is found to be

$\Delta\nu_{c\rightarrow d} \simeq -2b_3^p/\pi$. A measurement of this difference would provide a direct, clean, and accurate test of CPT for the proton.

CONCLUSIONS

In summary, we find that by using a general framework we are able to analyze proposed tests of CPT in hydrogen and antihydrogen. We find that in addition to testing CPT, these experiments will also test Lorentz symmetry. Our analysis shows that in comparisons of 1S-2S transitions in hydrogen and antihydrogen, control of the spin mixing at the 1S and 2S levels is an essential feature in designing an effective test of CPT and Lorentz symmetry. We also find that high-resolution radio frequency experiments in hydrogen or antihydrogen offer the possibility of new and precise tests of CPT and Lorentz symmetry. One very recent experiment using a double-resonance technique in a hydrogen maser has obtained a new CPT and Lorentz bound at the level of 10^{-27} for electrons and protons.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Foundation under grant number PHY-9801869.

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